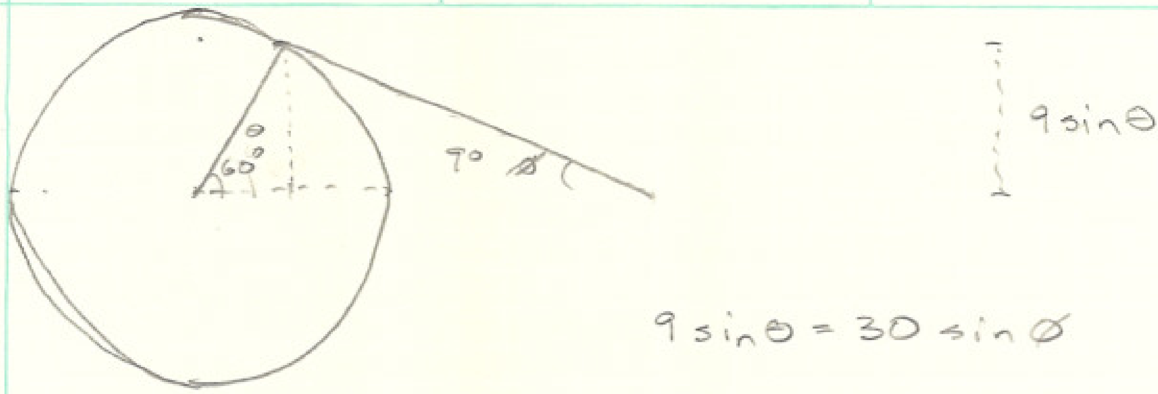


$$9\sin\theta = 30\sin\phi$$

$$\therefore \phi = 15.06^\circ$$

$$V_B = \dot{x}_B = -9\sin\theta\cdot\omega - 30\sin\phi\ \dot{\phi}$$

$$\therefore \dot{\phi} = 1.553\ \text{rad/s}$$

$$V_B = -90.05\ \text{in/s}$$

ASSIGNMENT #6

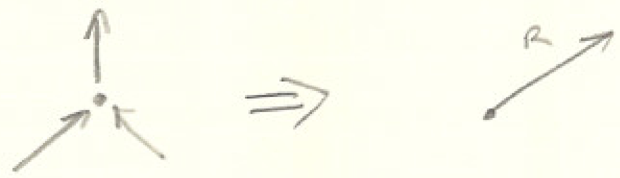
CH 14

1,4,5,23,26

CH 15

3,4,7,50,54

CHAPTER 15 KINETICS OF PARTICLES.



$\vec{R} = \Sigma F$

when equilibrium $\vec{R} = 0$

now we will combine static (force analysis) and the kinemetics (accereation)

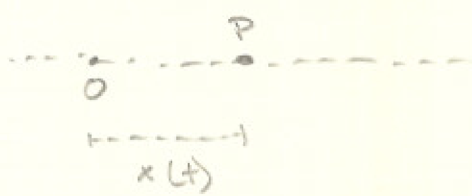
the relationship is

$\vec{R} = m\vec{a}$

force :	N	lb
mass :	kg.	slug.
acceleration :	m/s^2	ft/s^2

$N = kg \cdot m/s^2$ $\qquad lb = slug \cdot ft/s^2$

RECTILINEAR MOTION.

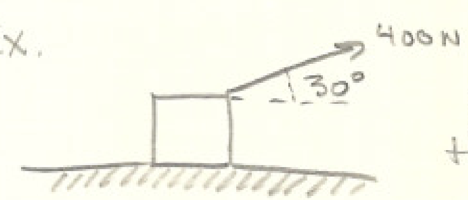


$v(t) = x'(t)$

$a(t) = v'(t) = x''(t)$.

$R_x = m a_x = m\ddot{x}$

EX.

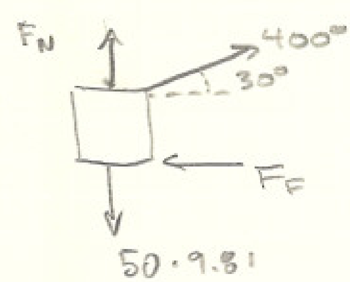


Determine the velocity after 3 s. the block starts at rest.

$m = 50\ kg$

$\mu_k = 0.3$

FBD.



$\Sigma F_y = 0 = F_N + 400 \sin 30° - 50 \cdot 9.81$

$\therefore F_N = 290.5 \text{ N}$

$\Sigma F_x = 400 \cos 30° - \mu \cdot F_N = 259.3 \text{ N}.$

$R_x = ma$

$259.3 = 50a$

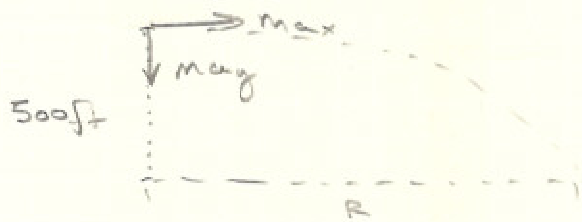
$\therefore a = 5.19 \text{ m/s}^2$

$v(t) = \int 5.19 \, dt = 5.19t$

$v(3) = 5.19(3) = 15.57 \text{ m/s}$

CURVILINEAR MOTION. (RECTANGULAR COORDINATE)

EX, A 30 lb projectile is fixed horizotally with an initial velocity of 750 ft/s from the top of the hill. Determine the range R of the projectile and the elasped time before it strikes the ground.



Solution:

$a_y = -32.2 \text{ ft/s}$ $\quad a_x = 0$

$v_y = \int a_y dt = -32.2t \text{ ft/s}$ $\quad v_y = 750 \text{ ft/s}$

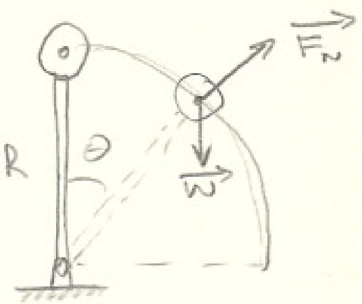
$x_x = \int v_y dt = -16.1t^2 + 500$ $\quad x_y = 750t$

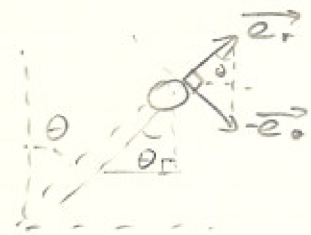
$x_x = 0$ then $t = 5.57 \text{ s}$ then $x_y = 4177.5 \text{ ft}.$

CURVILINEAR MOTION (POLAR COORDINATES)

EX. A sphere of mass m is attached to the top end of a slender verticle rod of neglectable mass. Determine the velocity v of the sphere and the force P in the rod when it hits the ground.

Given $m = 5\,kg$ $R = 2\,m$





$$\vec{a} = a_r \vec{e_r} + a_\theta \vec{e_\theta}$$

$$\Sigma F_r = P - 5g\cos\theta = 5a_r$$

$$\Sigma F_\theta = 5g\sin\theta = 5a_\theta$$

$$\begin{cases} a_r = \ddot{r} - r\dot{\theta}^2 \\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \end{cases}$$

$r = 2\,m$ $\dot{r} = \ddot{r} = 0$

$$a_r = 2\dot{\theta}^2$$

$$a_\theta = 2\ddot{\theta} = g\sin\theta$$

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta} \cdot \frac{d\theta}{dt} = \dot{\theta}\frac{d\dot{\theta}}{d\theta}$$

$$2\dot{\theta}\frac{d\dot{\theta}}{d\theta} = 9.81\sin\theta$$

$$2\int_0^{\dot\theta} \dot\theta \, d\dot\theta = \int_0^{\theta} 9.81 \sin\theta \, d\theta$$

$$\dot\theta^2 = 9.81(-\cos\theta)\Big|_0^{\theta} = 9.81(1-\cos\theta)$$

$$\dot\theta = \sqrt{9.81(1-\cos\theta)}$$

At $\theta = 90° = \pi/2$ $\quad \dot\theta = 3.132 \text{ rad/s}$

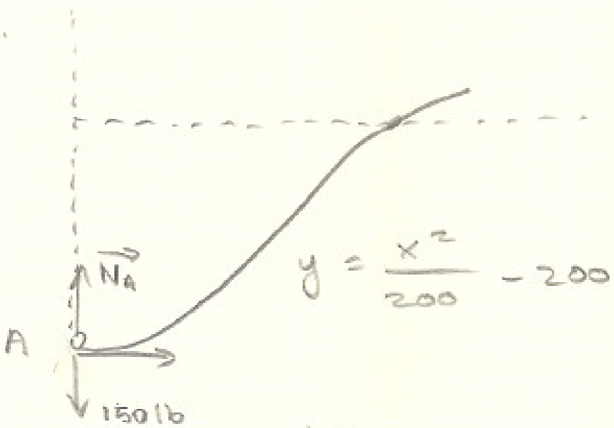
$V = r\dot\theta = 6.26 \text{ m/s}$

$$P = 5a_r + 5g\cos\theta$$
$$= 5(-2)(3.132)^2 + 5(9.81)\cos(90°)$$
$$= -98.1 \text{ N}$$

means the force is going this way.

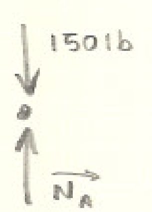
CURVILINEAR MOTION (NORMAL / TANGENTIAL COORDINATES)

EX.

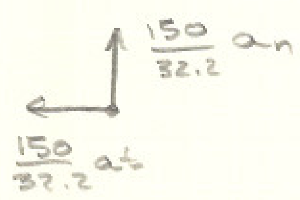


Determine the normal force on the 150 lb skier the instant he arrives at the end of the jump, where his velocity is 65 ft/s. Also, what is the acceleration at the point.

FBD.



KD



at the instant when the skier is at point A.

$$0 = \frac{150}{32.2} a_t$$

$$N_A - 150 = \frac{150}{32.2} a_n$$

remember that.

$$a_t = \dot{v}$$

$$a_n = \frac{v^2}{\rho}$$

$$\frac{1}{\rho} = \frac{|y''|}{(1 + y'^2)^{3/2}}$$

$$y' = \frac{x}{100}$$

$$y'' = \frac{1}{100}$$

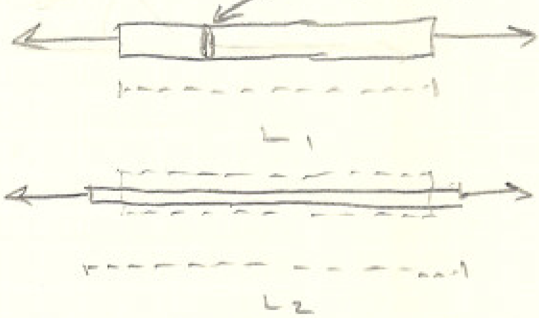
$$1/\rho = 1/100$$

$$a_n = \frac{v^2}{\rho} = 65^2 \frac{1}{100} = 42.2 \text{ ft/s}^2$$

MECHANICS OF MATERIALS

AXIAL LOAD.

AREA OF THE CROSS SECTION

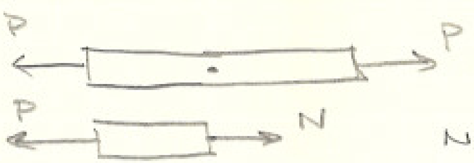


BODY BECOME DEFORMED.

to find the "k" constant between the force and the deformation. First we must introduce a few concepts.

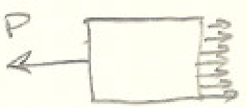
1. STRESS

the stress describes the intensity of the internal force on a specific plane passing through a point.

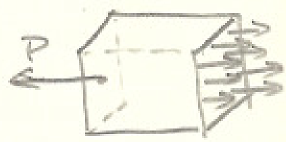


$N = P$

ENLARGE



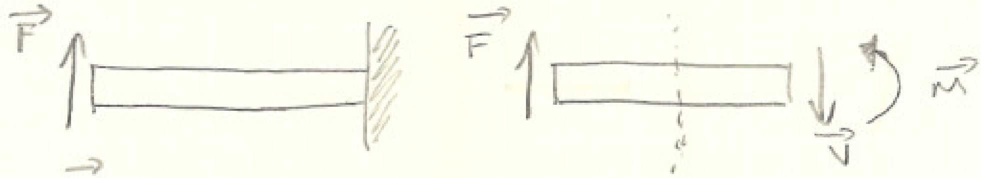
} parallel force system.

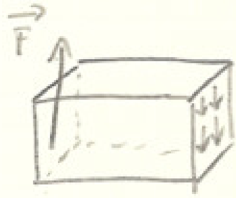


note: that if the cross section is far away from the ends, we assume uniform distribution

σ : intensity of the internal force (Normal Stress.)

$\sigma A = N \quad \therefore \quad \sigma = \dfrac{N}{A}$





if the cross section is far from both ends we again assume uniform distribution.

$\vec{V} = \tau A$

$\tau = \frac{\vec{V}}{A}$

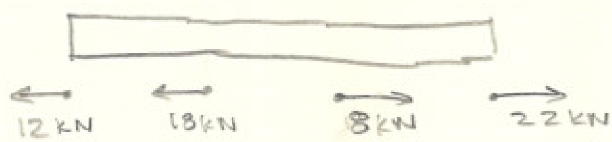
τ : shear stress.

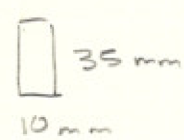
units for stress are $N/m^2 = Pa$.

$lb/in^2 = psi$

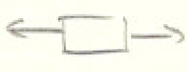
$k\,lb/in^2 = ksi$

EX. THE BAR HAS A CONSTANT WIDTH OF 35mm AND THICKNESS OF 10mm. DETERMINE THE MAX AVERAGE NORMAL STRESS IN THE BAR WHEN IT IS SUBJECTED TO LOADING.



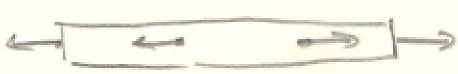


$= 0.0003500\, m^2$



$P = 12\,kN$

$P = 30\,kN$



$P = 22\,kN.$

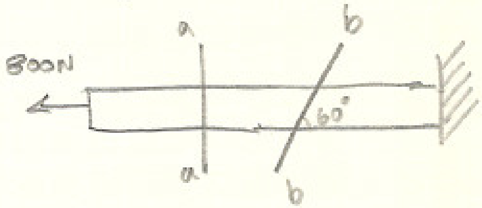
$\sigma_{max} = \frac{30 \times 10^3}{350 \times 10^{-6}} = 85.7\,MPa.$

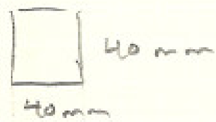
EX.

The bar has a cross section. Determine the normal and shear stress acting on the bar along.

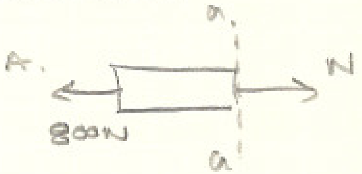
A. plane a – a

B. plane b – b.



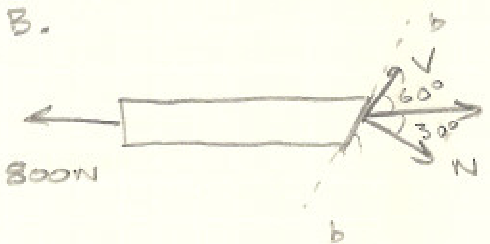


SOLUTION:



$$\sigma = \frac{N}{A} = \frac{800\ N}{(0.04)^2} = 500\ kPa.$$

$$\tau = 0$$



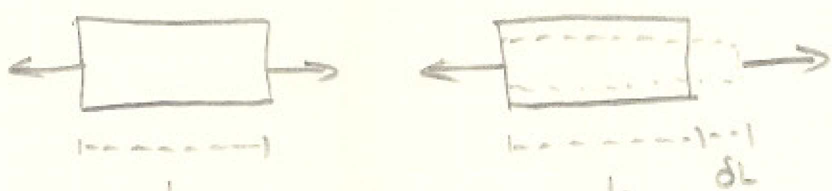
N = 692.8 N

V = 400 N.

$$\sigma = \frac{N}{A} = \frac{692.8}{(0.04)^2 / \sin 60^\circ} = 374.99\ kPa.$$

$$\tau = \frac{V}{A} = \frac{400}{(0.04)^2 / \sin 60^\circ} = 217\ kPa.$$

2. STRAIN.



normal strain: the elongation or contraction of a line segment per unit of length.

$$\epsilon = \frac{\delta L}{L} \quad (m/m) \quad (in/in)$$

$\epsilon > 0$: will elongate

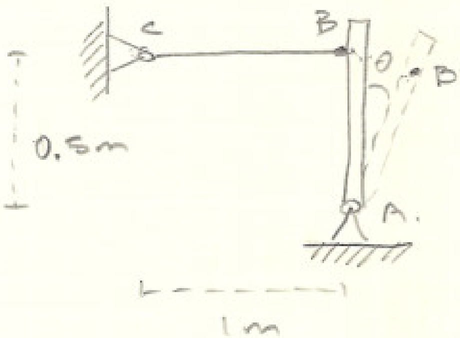
$\epsilon < 0$: will contract

SHEAR STRAIN the change in angle that the two lines that we originally perpendicular to one another.

cross section

$$\gamma_{nt} = \pi/2 - \theta'$$

EX.



$\theta = 0.002$ rad.

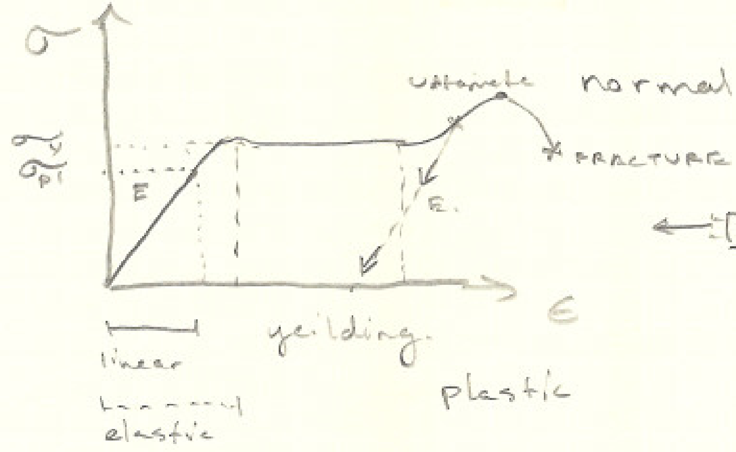
Determin the normal strain in the cable BC.

$$\epsilon = \frac{B'C - BC}{BC}$$

$$= \frac{B''B}{BC} = 0.001 \ m/m$$

- HOOKE'S LAW.

normal stress vs. strain.



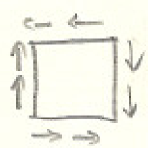
yeilding: the bar will continue to i/c length but it will not increase internal stress.

plastic: once the strain reaches this pt. it is permanetely deformed. and will return down the E slope.

E: is called the stiffness, modulus of Elasticity. or Young's modulus.

$$\sigma = E\epsilon$$

SHEAR STRESS VS. SHEAR STAIN.



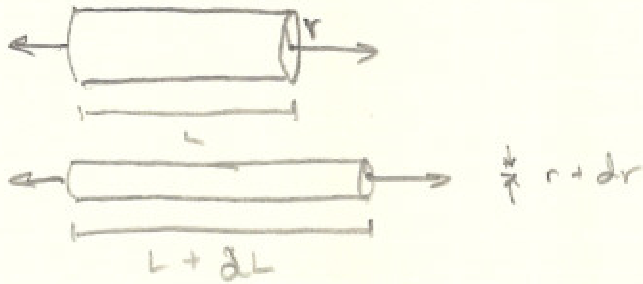
SHEAR STRESS τ

SHEAR STRAIN γ

$$\tau = G\gamma$$

G: shear modulus of elasticity.

• POISSEN'S RATIO.



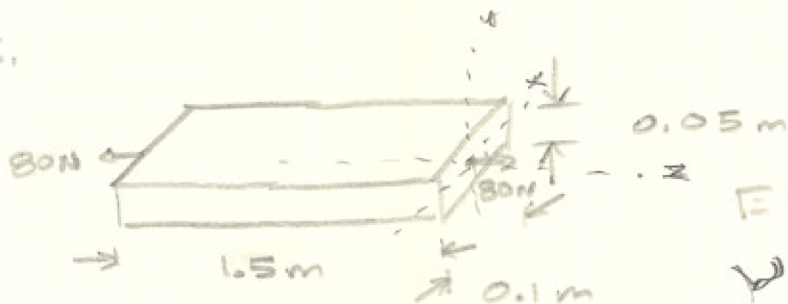
$$\epsilon_{LONG} = \frac{dL}{L} \qquad \epsilon_{LAT} = \frac{dr}{r}$$

$$\nu = -\frac{\epsilon_{LATER}}{\epsilon_{LONG}}$$

note: for most material $\frac{1}{4} \leq \nu \leq \frac{1}{3}$

$$G: \frac{E}{2(1+\nu)}$$

EX.



$E = 200$ GPa.

$\nu = 0.32$.

Determine the change in it's length and it's change in dimesions of it's cross section

SOLUTION:

$$\epsilon_z = \frac{dL}{L} \qquad dL = \epsilon_z L$$

$$\sigma_z = \frac{80\,kN}{(0.1)(0.05)} = 16.0 \times 10^6 \text{ pa.}$$

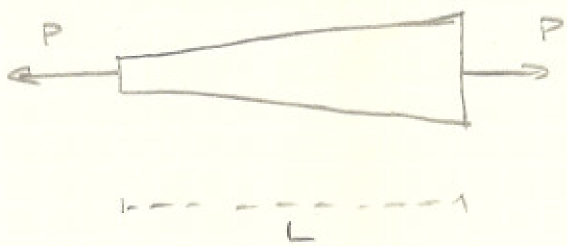
$$\epsilon_z = \frac{\sigma_z}{E} = \frac{16.0 \times 10^6}{200 \times 10^9} = 80 \times 10^{-6} \text{ m/m}$$

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = \frac{-\epsilon_y}{\epsilon_z}$$

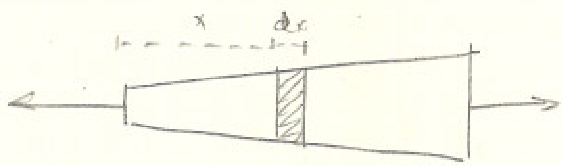
$$\epsilon_x = \epsilon_y = (\nu)(-\epsilon_z)$$
$$= 0.32 \cdot 8 \times 10^{-6}$$
$$= -25.6 \times 10^{-6}$$

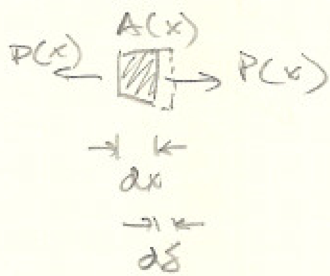
$$d_x = 0.1 \times (-25.6 \times 10^{-6}) = -2.56(10^{-6})\ m$$
$$d_y = 0.05 \times (-25.6 \times 10^{-6}) = -1.28 \times 10^{-6}\ m.$$

• AXIAL LOAD.



Ex. find the realitive displacement of one end of the bar in relation to the other end.





$$\therefore \quad \sigma(x) = \frac{P(x)}{A(x)}$$

$$d\delta = \epsilon_x \, dx$$

$$\therefore \quad \epsilon_x = \frac{\sigma(x)}{E} = \frac{P(x)}{EA(x)}$$

$$d\delta = \frac{P(x)}{EA(x)} dx$$

$$\delta = \int d\delta = \int_0^L \frac{P(x)}{EA(x)}\,dx$$

in the special case where $A(x)$ & $P(x)$ are constant.

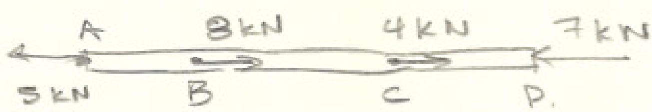
$$\delta = \frac{PL}{EA}$$

now lets take a look.

$$\delta = \frac{P}{\frac{EA}{L}} \qquad \text{let } k = \frac{EA}{L}$$

$$\delta = \frac{P}{k} \qquad k \text{ is the strain constant.}$$

EX.



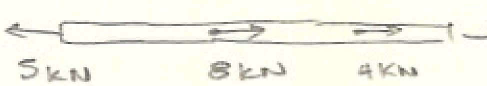
$A = 30\ mm^2$

$E = 200$ GPa.

find $\delta_{A/D}$.

$P_{AB} = 5$ kN

$P_{BC} = -3$ kN



$= -7$ kN.

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}.$$

$$= \frac{P_{AB}L_{AB}}{EA} + \frac{P_{BC}L_{BC}}{EA} + \frac{P_{CD}L_{CD}}{EA}.$$

$$= \frac{5(10^3)(1)}{200(10^9)(30)(10^{-6})} + \frac{-3(10^3)(1.5)}{200(10^9)(30)10^{-6}} + \frac{-7(10^3)(0.5)}{200(10^9)(30)(10^{-6})}$$